



**D&D Resources Ltd**  
Mathematics Exam Preparation Made Easy

External Achievement Standard - 91586

# Practice External Assessments 3

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NCEA 3 Statistics

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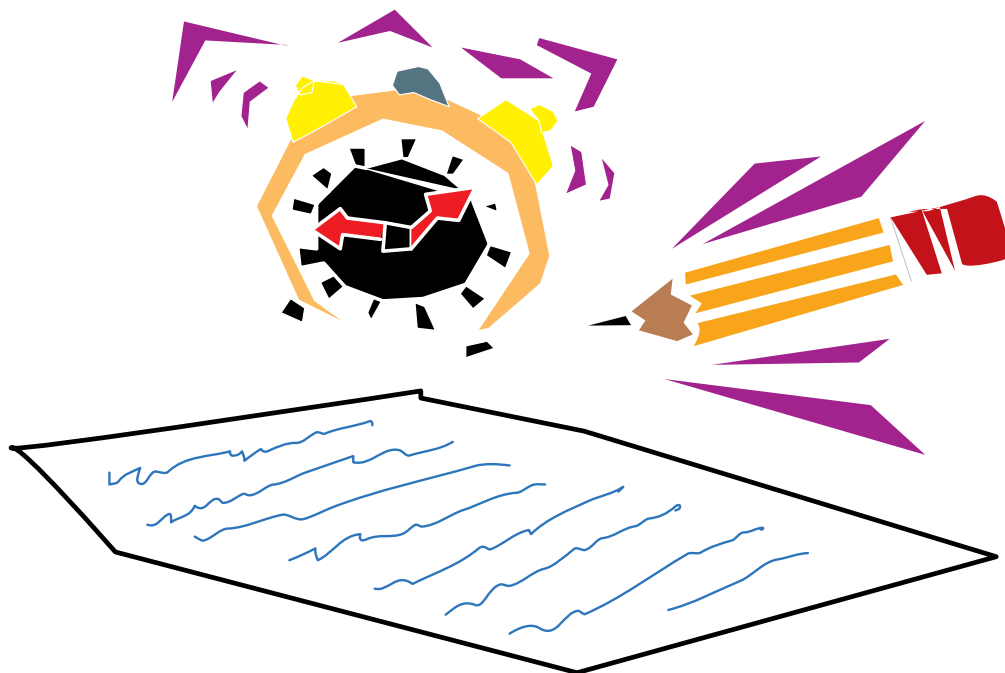
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The questions in the practice assessments are NOT in order of difficulty. Attempt all questions or you may not provide enough evidence to achieve the required standard.

## Achievement Standard

91586

Apply Probability Distributions in Solving Problems

Achievement	Achievement with Merit	Achievement with Excellence
<ul style="list-style-type: none"> <li>Apply probability distributions in solving problems.</li> </ul>	<ul style="list-style-type: none"> <li>Apply probability distributions, using relational thinking, in solving problems.</li> </ul>	<ul style="list-style-type: none"> <li>Apply probability distributions, using extended abstract thinking, in solving problems.</li> </ul>

- ◆ This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
  - ❖ Investigate situations that involve elements of chance
    - calculating and interpreting expected values and standard deviations of discrete random variables.
    - applying distributions such as the Poisson, binomial, and normal.
- ◆ Apply probability distributions in solving problems involves:
  - ❖ selecting and using methods
  - ❖ demonstrating knowledge of concepts and terms
  - ❖ communicating using appropriate representations.
- ◆ Relational thinking involves one or more of:
  - ❖ selecting and carrying out a logical sequence of steps
  - ❖ connecting different concepts or representations
  - ❖ demonstrating understanding of concepts;
 and also relating findings to a context, or communicating thinking using appropriate statements.
- ◆ Extended abstract thinking involves one or more of:
  - ❖ devising a strategy to investigate or solve a problem
  - ❖ identifying relevant concepts in context
  - ❖ developing a chain of logical reasoning
  - ❖ making a generalisation;
 and also, where appropriate, using contextual knowledge to reflect on the answer.
- ◆ Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or statistical contexts.
- ◆ Methods include a selection from those related to:
  - ❖ discrete and continuous probability distributions
  - ❖ mean and standard deviation of random variables
  - ❖ distribution of true probabilities versus distribution of model estimates of probabilities versus distribution of experimental estimates of probabilities.

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(a) (i) Calculate the number of warranty claims Elisa can expect in any given week.

w	0	1	2	3	4
P(W = w)	0.78	0.13	0.06	0.02	0.01

(ii) Explain your answer from part (i) above in terms of the context of the question.

- Using your result from part (a) and an appropriate distribution to model this situation, calculate the probability that Elisa receives no more than two warranty claims within a four week period.

- (ii) Justify your choice of probability distribution for your answer in part i) and state any assumptions you need to make.

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PEA 3

You are advised to spend 60 minutes answering this assessment.

You should show ALL working and answer ALL parts of ALL questions.

### QUESTION ONE

- (a) Alec is an insurance salesman. He sells life insurance policies to 6 men all of the same age and in excellent health. According to actuarial tables the probability that a man of this age will be alive in 30 years time is 0.4.

Assuming each man's life expectancy is independent of the other calculate the probability that at least 3 of the six men will be alive in 30 years.

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Alec's travel time to work can be assumed to be normally distributed with a mean of 26 minutes and a standard deviation of 6.4 minutes.

- (b) If Alec starts work at 8.30 am find the probability on a particular day, assuming he leaves home at 8.00 am, that he will be late for work.

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- (c) Alec's boss has noticed that in the last month he has been late to work 40% of the time.

Assuming he leaves home at 8.00 am and the standard deviation for his travel time is 6.4 minutes, what is the average time it has taken Alec to travel to work in the last month?

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## QUESTION TWO

- (a) A dentist keeps records on the number of visits made by his patients in a 12 month period.

The table below shows the probability distribution of the random variable  $X$ , the number of visits made by patients to the dentist in a 12 month period.

$x$	0	1	2	3	4	5
$P(X = x)$	0.15	0.47	0.18	0.09	0.07	0.04

- (i) Calculate the mean number of visits made by patients to the dentist in a 12 month period.

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- (ii) The cost of a single consultation with the dentist is \$85.00. Patients pay for the consultation and often purchase, while there, other dental products, such as dental floss, brushes, piksters etc. If the mean amount paid by patients in a 12 month period is \$148.00, how much do they spend on 'other' dental products?

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- (iii) Let the random variable  $Y$  represent the number of visits made by the patients of another dentist in a different part of the city in a 12 month period. The random variable  $Y$  has  $SD(Y) = 0.95$ .

Show that  $X$  has a larger standard deviation than  $Y$  and give ONE reason why this may be the case.

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- (b) The number of phone calls received at the dentist's office is thought to follow a Poisson distribution, with a mean number of 4.8 calls per hour.

- (i) Using this model calculate the probability that there are more than 12 calls in a three-hour period.

Identify any assumption(s) you need to make.

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# Answers – 91586

# PEA 3

Quest.	Evidence	Achievement	Merit	Excellence
		Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.
ONE		TWO of:	TWO of:	ONE of:
(a)	Binomial distribution, $n = 6$ , $\pi = 0.40$ . $P(X \geq 3) = 0.4557$ (calculator)	• correct probability.		
(b)	Normal distribution, $\mu = 26$ , $\sigma = 6.4$ $P(X > 30) = P(Z > \frac{30-26}{6.4})$ $= P(Z > 0.625)$ $= 0.2660$ (calculator)	• correct probability.		
(c)	Inverse normal $P(X > 30) = 0.40$ $P(Z > \frac{30-\mu}{6.4}) = 0.253$ $\frac{30-\mu}{6.4} = 0.253$ $\mu = 28.4$ minutes	• identifies inverse normal and $P(X > x) = 0.40$	• mean correctly calculated.	
(d) (i)	X = Binomial distribution Fixed number of client contacts ie. 720. Each client Alec sees or doesn't see i.e. two possible outcomes. Each client contact is independent of the other. The probability of a client contacting Alec is fixed i.e. 0.85.	• identifies binomial distribution and parameters.	• identifies binomial distribution and parameters AND justifies choice of distribution in context.	
(d) (ii)	Use the normal distribution because the binomial distribution can approximate the normal distribution when n is large. Parameters: mean = $720 \times 0.85 = 612$ std. dev. = $\sqrt{720 \times 0.85 \times 0.15} = 9.58$ Continuity correction. $P(X > 620.5)$ $= P(Z > \frac{620.5 - 612}{9.58})$ $= P(Z > 0.887)$ $= 0.1875$ (calculator)	• identifies Normal distribution and justifies that the normal distribution can approximate the binomial when n is large.	• identifies Normal distribution and justifies that the normal distribution can approximate the binomial when n is large AND identifies parameters.	• identifies Normal distribution and justifies that the normal distribution can approximate the binomial when n is large AND identifies parameters AND calculates correct probability using a continuity correction.