D&D Resources Ltd
Mathematics Exam Preparation Made Easy

Integration - 91579

Practice External Assessments 3

Integration

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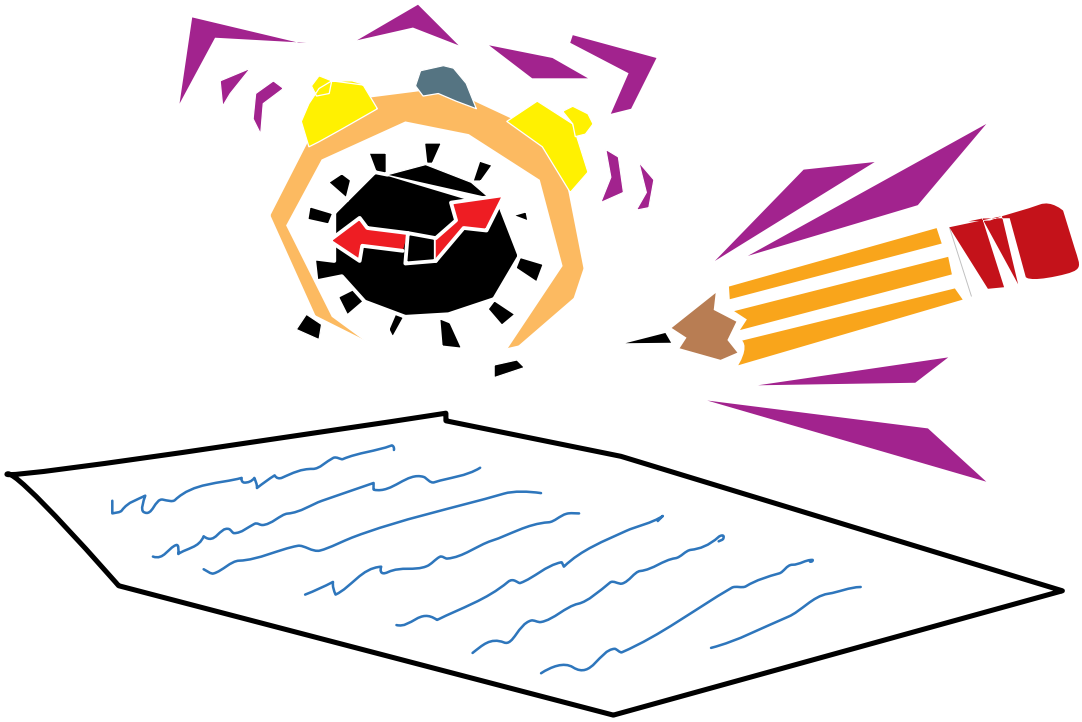
Cover design:
D & D Resources Ltd.
Image: Stained Glass Texture @ BackgroundStor

Printed in New Zealand by
Format Print, Wellington

ISBN 978 0 9922606 2 0

NCEA 3 Maths

Contents



Prior to attempting the practice assessments we suggest you download our Integration, 'Readiness Check', from the D & D Resources website. It comprises a series of questions together with answers which you can use as a 'warm up' before undertaking the assessments in this booklet.

The questions in the practice assessments are NOT in order of difficulty. Attempt all questions or you may not provide enough evidence to achieve the required standard.

Achievement Standard

91579 **Applying integration methods in solving problems**

Achievement	Achievement with Merit	Achievement with Excellence
• Apply integration methods in solving problems.	• Apply integration methods, using relational thinking, in solving problems.	• Apply integration methods, using extended abstract thinking, in solving problems.

- This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
  - choose and apply a variety of integration and anti-differentiation techniques to functions and relations using both analytical and numerical methods
  - form differential equations and interpret the solutions

  in the Mathematics strand of the Mathematics and Statistics Learning Area.
- Apply integration methods in solving problems involves:
  - selecting and using methods
  - demonstrating knowledge of concepts and terms
  - communicating using appropriate representations.
- Relational thinking involves one or more of:
  - selecting and carrying out a logical sequence of steps
  - connecting different concepts or representations
  - demonstrating understanding of concepts
  - forming and using a model;

  and relating findings to a context, or communicating thinking using appropriate mathematical statements.
- Extended abstract thinking involves one or more of:
  - devising a strategy to investigate or solve a problem
  - identifying relevant concepts in context
  - developing a chain of logical reasoning, or proof
  - forming a generalisation;

  and using correct mathematical statements, or communicating mathematical insight.
- Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.
- Methods include a selection from those related to:
  - integrating power, polynomial, exponential (base e only), trigonometric, and rational functions
  - reverse chain rule, trigonometric formulae
  - rates of change problems
  - areas under or between graphs of functions, by integration
  - finding areas using numerical methods, e.g. the rectangle or trapezium rule
  - differential equations of the forms $y' = f(x)$ or $y'' = f(x)$ for the above functions or situations where the variables are separable (e.g. $y' = ky$) in applications such as growth and decay, inflation, Newton's Law of Cooling and similar situations.

91579 **PEA 2**

You are advised to spend 60 minutes answering this assessment.

You should show ALL working and answer ALL parts of ALL questions.

QUESTION ONE

(a) Find $\int \pi + e^{\pi x}\ dx$.

(b) Find the value of k if $\int_1^2 \left(x^3 - \frac{k}{x^3}\right) dx = 0$

(c) A train's velocity, V, t hours after leaving a railway station is given by the formula

$$V = \frac{100t}{\sqrt{t^2+1}}$$ where V is in km/h.

How far does the train travel in the first two hours after leaving the station?

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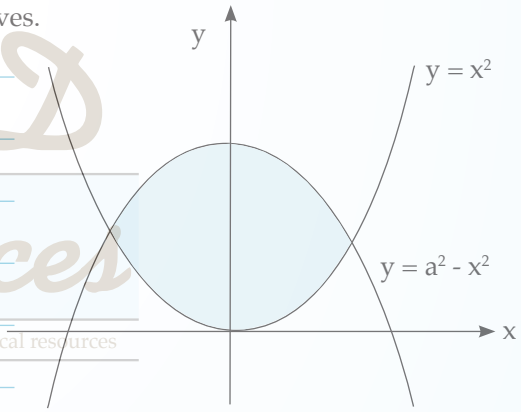
(d) Milk leaks from a vat so that the rate of change of the depth, h, of the milk, in metres, is modelled by the differential equation $\frac{dh}{dt} = \frac{-h^2}{5}$, where t is the time in minutes. The initial depth of milk in the vat is 1.5 m. Solve the differential equation to find out how long it takes for the depth of milk in the vat to decrease to 10 cm.

(e) The graph below shows the curves $y = a^2 - x^2$ and $y = x^2$.
Find, in terms of a, the shaded area between the two curves.



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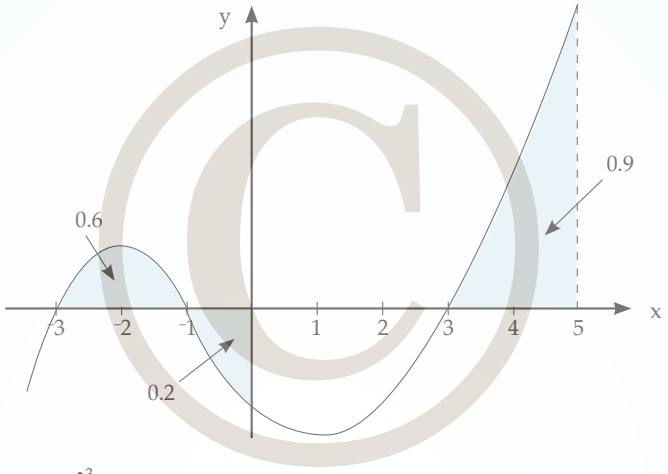
QUESTION TWO

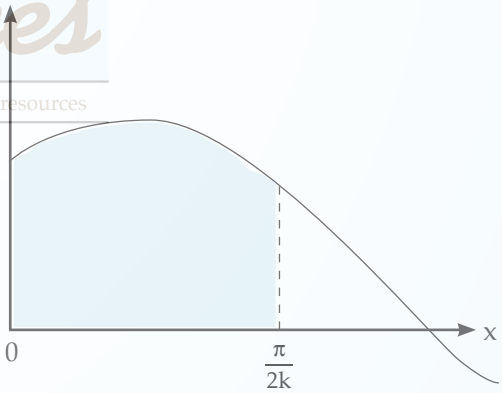
(a) Find $\int(5x^2 - \frac{5}{2x+1})dx$.

(b) The graph of the function y = f(x) is shown below. The areas of the shaded regions are given.



If $\int_{-3}^{5} f(x)dx = 0.5$, what is $\int_{0}^{3} f(x)dx$?

(c) Find an expression in terms of k for the area between the curve y = cos kx + sin kx and the x axis from x = 0 to x = $\frac{\pi}{2k}$.



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Answers – 91579 PEA 6

TWO		TWO of:	ONE of:	ONE of:
(a)	$\frac{5x^3}{3}-\frac{5\ln(2x+1)}{2}+C$	• correct integration.		
(b)	$\int_0^3 f(x)dx = {}^-0.8$ $(0.6+0.9-0.2-0.8=0.5)$	• correct solution.		
(c)	$\text{Area}=\int_0^{\pi/2k}\cos kx+\sin kx\,dx$ $=\left[\frac{\sin kx}{k}-\frac{\cos kx}{k}\right]_0^{\pi/2k}$ $=\left[\frac{\sin\frac{\pi}{2}}{k}-\frac{\cos\frac{\pi}{2}}{k}\right]-\left[\frac{\sin 0}{k}-\frac{\cos 0}{k}\right]$ $=\left[\frac{1}{k}-0\right]-\left[0-\frac{1}{k}\right]$ $=\frac{2}{k}$	• correct integration.	• correct integration and solution in terms of k.	
(d)	$\frac{dy}{\sqrt{y+2}}=\frac{1}{x}dx$ $2\sqrt{y+2}=\ln x+C$ At $x=1, y=7$ and $C=6$ $2\sqrt{y+2}=\ln x+6$ At $y=10, x=2.53$	• correct general solution of DE.	• correct DE and solution.	
(e)	$\frac{dX}{X-c}=kdt$ $\ln\lvert X-c\rvert=kt+C$ $X-c=Ae^{kt}$ $X=Ae^{kt}+c$ At $t=0, X=0, A={}^-c$ $X=c-ce^{kt}$, and at $t=0, c=C_0$ $X=C_0(1-e^{kt})$ $\frac{X}{C_0}=1-e^{kt}$ $e^{kt}=1-\frac{X}{C_0}$ $kt=\ln\left(1-\frac{X}{C_0}\right)$ $t=\frac{\ln\left(1-\frac{X}{C_0}\right)}{k}=\frac{1}{k}\ln\left(\frac{C_0-X}{C_0}\right)$	• correct general solution of DE.	• correct general solution and expression using C_0.	• correct solution.

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