Complete the following questions on the next four pages as a 'warm up' before attempting the six practice achievement standards in our PEA 3 Integration Booklet.

QUESTION ONE

Find the integrals. Remember any arbitrary constants.

(a) $\int (6x^3 - \frac{3}{x^2})\ dx$

(b) $\int (2x + \frac{2}{\sqrt{x}})\ dx$

(c) $\int 4(3x+2)^3\ dx$

(d) $\int 2e^{5x-1}\ dx$

(e) $\int \sqrt[4]{8x-1}\ dx$

(f) $\int \frac{6}{2x+3}\ dx$

(g) $\int 3\sin 2x\ dx$

(h) $\int {}^{-}\operatorname{cosec} 6x \cot 6x\ dx$

(i) $\int \frac{9x+2}{x}\ dx$

QUESTION TWO

(a) An object moves in a straight line so that after t seconds, its acceleration in ms^{-2} is given as $a = 8\sin 2t$. If the velocity of the object after $\frac{\pi}{2}$ seconds is 7 ms^{-1}, find its distance s, from its initial position at that moment.

(b) The acceleration in ms^{-2} of a particle is given by the equation $a = \frac{{}^{-}1}{\sqrt{t+1}}$ where t is measured in seconds. If the initial velocity of the particle is 2 ms^{-1}, at what time is the particle stationary?

QUESTION THREE

(a) Solve the differential equation $\frac{dy}{dx} = \frac{2x}{e^y}$ given that $y = 0$ when $x = 2$.

QUESTION THREE cont...

(b) If $\frac{dy}{dx} = \frac{(x+1)^2}{2y}$ and $y = 4$ when $x = 2$, find y when $x = 8$.

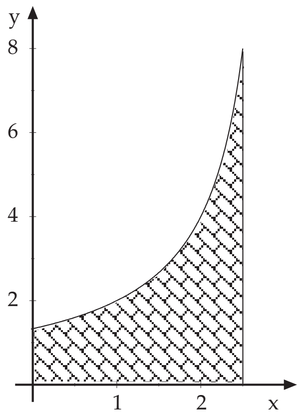
(c) Find the general solution to the differential equation $\frac{dy}{dx} = kxy$.

QUESTION FOUR

(a) A backyard has a lawn and a paved area. The paved area in m^2 can be modelled by the shaded area on the graph below.

The shaded area is bounded by $y = \frac{4}{3-x}$, $x = 0$ and $x = 2.5$.

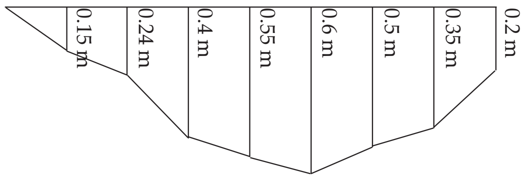
Calculate the area of the paving.



(b) Find the area enclosed between the graph $y = \sqrt[3]{3x+1} + 1$, the x axis and the lines $x = 0$ and $x = 4$.

QUESTION FIVE

(a) The cross sectional area of a stream is found by finding the depths at equal intervals from one bank to another. The stream is 4 metres wide. Use Simpson's Rule to calculate the cross-sectional area of the stream.



(b) Use the table below to find an approximation to $\int_1^3 \sqrt{\ln x}$ using the Trapezium Rule.

x	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0
$\sqrt{\ln x}$	0	0.47	0.64	0.75	0.83	0.90	0.96	1.00	1.05

QUESTION SIX

Find the integrals.

(a) $\int 6\sin 5x \sin 3x \; dx$

(b) $\int \frac{x}{x^2-5} \; dx$

(c) $\int \frac{3x-2}{x-1} \; dx$

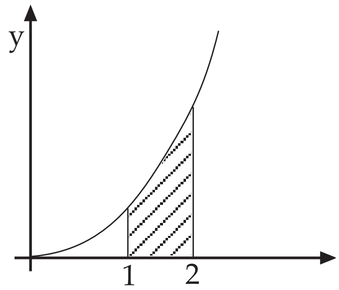
(d) $\int 2x^3(x^4+3)^2 dx$

(e) $\int \cot^2 x \; dx$

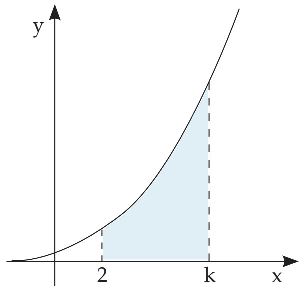
QUESTION SEVEN

(a) Below is a sketch of the graph $y = x e^{x^2}$.

Find the area between the curve and the x axis from $x = 1$ to $x = 2$.



(b) The graph shows the function $y = \left(\frac{x}{2}+1\right)^3$ and the lines $x = 2$ and $x = k$, where $k > 2$. Find the value of k so that the area between the curve and the x axis is 40.



QUESTION EIGHT

A radioactive substance is disintegrating at the rate of $0.5e^{-t/25}$ grams per year.

How much of the substance disintegrates during the first 50 years?

QUESTION NINE

Use integration to find the area enclosed between the graphs of the functions $y = x(6 - x)$ and $y = {}^{-}x + 10$.

QUESTION TEN

Find the value of k such that $\int_2^k \frac{4x+k}{2x^2+kx} = \ln 3$.

QUESTION ELEVEN

A small rural town has experienced a decline in population since 2005. At the end of that year, there were 3900 residents and at the end of 2015, 3220 residents. If the rate of change of the population is proportional to the population, write an equation for the population N in terms of time t years since 2005 and use it to predict the population at the end of 2020.

QUESTION TWELVE

A cup of tea is made with boiling water at a temperature of 100˚ C, in a kitchen with room temperature of 24˚ C. The rate at which the temperature of the tea changes at any instant is proportional to the difference between the temperature of the tea and the room temperature at that instant. After 3 minutes, it has cooled to 70˚C. How long will it take for the temperature to drop below 40˚ C and the tea to taste cold?

Answers – Readiness Check

Question One:

(a) $\frac{3}{2}x^4 + \frac{3}{x} + C$

(b) $x^2 + 4\sqrt{x} + C$

(c) $\frac{(3x+2)^4}{3} + C$

(d) $\frac{2e^{5x-1}}{5} + C$

(e) $\frac{(8x-1)^{5/4}}{10} + C$

(f) $3 \ln|2x + 3| + C$

(g) $\frac{^-3\cos 2x}{2} + C$

(h) $\frac{\operatorname{cosec} 6x}{6} + C$

(i) $9x + 2\ln|x| + C$

Question Two:

(a) $V = \frac{^-8\cos 2t}{2} + C$

$V = {^-4}\cos 2t + C$

When $V = 7$, $C = 3$

$V = {^-4}\cos 2t + 3$

$S = {^-2}\sin 2t + 3t$

at $t = \frac{\pi}{2}$, $S = 4.71$ m (2 dp)

(b) $v = {^-2}\sqrt{t+1} + c$

At $t = 0$, $v = 2$ and $c = 4$

Particle is stationary when $v = 0$.

${^-2}\sqrt{t+1} + 4 = 0$

$t = 3$

Question Three:

(a) $e^y dy = 2x dx$

$e^y = x^2 + c$

when $y = 0$, $x = 2$ and $c = {^-3}$

$e^y = x^2 - 3$

$y = \ln|x^2 - 3|$

(b) $2y dy = (x+1)^2 dx$

$y^2 = \frac{(x+1)^3}{3} + c$

At $x = 2$, $y = 4$ and $c = 7$

$y^2 = \frac{(x+1)^3}{3} + 7$

At $x = 8$, $y = 5\sqrt{10}$ or 15.8

(c) $\frac{dy}{y} = kx dx$

$\ln|y| = \frac{kx^2}{2} + C$

$y = Ae^{kx^2/2}$

Question Four:

(a) $\text{Area} = \int_0^{2.5} \frac{4}{3-x} dx$

$= [{^-4}\ln|3-x|]_0^{2.5}$

$= 7.17 \text{ m}^2$

(b) $\text{Area} = \int_0^4 \sqrt[3]{3x+1} + 1 dx$

$= \left[\frac{(3x+1)^{4/3}}{4} + x\right]_0^4$

$= 11.4$

Question Five:

(a) $\int_0^8 f(x)dx = \frac{0.5}{3}[0 + 0.2 + 4(0.15 + 0.4 + 0.6 + 0.35) + 2(0.24 + 0.55 + 0.5)]$

$= 1.46 \text{ m}^2$

(b) $\int_1^3 \sqrt{\ln x} = \frac{0.25}{2}[0 + 1.05 + 2(0.47 + 0.64 + 0.75 + 0.83 + 0.90 + 0.96 + 1.00)]$

$= 1.52$

Question Six:

(a) $\frac{3\sin 2x}{2} - \frac{3\sin 8x}{8} + C$

(b) $\frac{1}{2}\ln|x^2 - 5| + C$

(c) $3x + \ln|x - 1| + C$

(d) If $y = (x^4 + 3)^3$ then

$\frac{dy}{dx} = 3(x^4+3)^2.4x^3 = 12x^3(x^4+3)^2$

$\int 2x^3(x^4+3)^2 dx = \frac{(x^4+3)^3}{6} + C$

(e) $\int \cot^2 x\, dx = \int \operatorname{cosec}^2 x - 1\, dx$

$= {^-\cot} x - x + c$

Question Seven:

(a) $\text{Area} = \int_1^2 xe^{x^2} dx$

$= \left[\frac{e^{x^2}}{2}\right]_1^2$

$= 25.94 \text{ units}^2$

(b) $\int_2^k \left(\frac{x}{2} + 1\right)^3 dx = \left[\frac{(0.5x+1)^4}{2}\right]_2^k$

$\frac{(0.5k+1)^4}{2} - 8 = 40$

$(0.5k+1)^4 = 96$

$k = 4.26$

Answers – Readiness Check

Question Eight: $\dfrac{dr}{dt} = 0.5e^{-t/25}$

$$r = \int_0^{50} 0.5e^{-t/25}$$

$$= \left[{}^{-}12.5e^{-t/25}\right]_0^{50} = 10.8 \text{ g}$$

Question Nine: $x(6 - x) = {}^{-}x + 10$

$x^2 - 7x + 10 = 0$

$x = 2, 5$

$$\text{Area} = \int_2^5 (7x - x^2 - 10)\,dx$$

$$= \left[\frac{7x^2}{2} - \frac{x^3}{3} - 10x\right]_2^5$$

$$= 4.5$$

Question Ten: $\left[\ln\left|2x^2 + kx\right|\right]_2^k = \ln 3$

$\ln\left|3k^2\right| - \ln\left|8 + 2k\right| = \ln 3$

$\dfrac{3k^2}{8 + 2k} = 3$

$3k^2 - 6k - 24 = 0$

$k = 4, {}^{-}2$

Only solution is $k = 4$ since $k > 2$.

Question Eleven: $\dfrac{dN}{dt} = kN$

$\dfrac{dN}{N} = kdt$

$\ln N = kt + C$

$N = Ae^{kt}$

When $t = 0$, $N = 3900$
so $A = 3900$

When $t = 10$, $N = 3220$

$3220 = 3900\, e^{kt}$ and $k = {}^{-}0.0191595...$

$N = 3900\, e^{-0.0191595t}$

At $t = 15$, $N = 2925$ or 2926

Question Twelve: $\dfrac{dT}{dt} = k(T - T_0)$

$\dfrac{dT}{(T - T_0)} = kdt$

$\ln(T - T_0) = kt + c$

$T - T_0 = Ae^{kt}$

When $t = 0$, $T = 100$, $T_0 = 24$ and $A = 76$

$T - 24 = 76e^{kt}$

At $t = 3$, $T = 70$

$46 = 76e^{3k}$, $k = {}^{-}0.1673639813$

$T - 24 = 76e^{-0.167364t}$

At $T = 40$, $16 = 76e^{-0.167364t}$

$t = 9.3$ minutes