

Complete the following questions on the next two pages as a 'warm up' before attempting the six practice achievement standards our PEA 3 Complex Numbers Booklet.

QUESTION ONE

$$u = -2 + i \text{ and } v = 3 - 2i$$

- (a) Write u to polar form.

- (b) Write v to polar form.

- (c) Write $u - v$ in rectangular form.

- (d) Write $3\bar{u} + v$ in rectangular form.

- (e) Write uv in rectangular form.

- (f) Write $\frac{v}{u}$ in rectangular form.

QUESTION TWO

$$w = 2 \operatorname{cis} \frac{3\pi}{8} \text{ and } z = 5 \operatorname{cis} \frac{-\pi}{6}$$

- (a) Write z in rectangular form.

- (b) Write wz in polar form.

- (c) Write $\frac{w}{z}$ in polar form.

- (d) Write w^{12} in rectangular form.

QUESTION THREE

Find the modulus and argument for:

(a) $z = 2 + 2i$

$$|z| = \quad \arg z = \quad$$

(b) $z = \sqrt{3} - i$

$$|z| = \quad \arg z = \quad$$

QUESTION FOUR

$$p = 4 - 3\sqrt{2} \text{ and } q = 2 + \sqrt{2}$$

- (a) Write pq in the form $p + q\sqrt{2}$.

- (b) Write $\frac{p}{q}$ in the form $p + q\sqrt{2}$.

QUESTION FIVE

- (a) Simplify $(\sqrt{18} - \sqrt{8})^2$

- (b) Simplify $\frac{\sqrt{27k^2}}{\sqrt{6k} \times \sqrt{2k}}$.

QUESTION SIX

- (a) Solve the equation $2\sqrt{x+2} = 2x$.

- (b) Solve the equation $\sqrt{x+4} = \sqrt{x} - k$.

QUESTION SEVEN

- (a) Solve $z^2 - 8z + 22 = 0$ and write your answer in the form $z = a \pm b\sqrt{c}i$.

- (b) Find the remainder when $x^3 + 3x^2 - 5x + 2$ is divided by $x + 4$.

- (c) If $x - 1$ is a factor of $2x^3 - x^2 + 3x + k = 0$, find the value of k .

- (d) One of the roots of the equation $z^3 - 6z^2 + 10z + k = 0$ where k is real is $1 + i$. Find the value of k and the other two roots.

QUESTION EIGHT

- (a) Find all solutions, real and complex of $z^4 = -36$. Write your answers in polar form.

- (b) Find all solutions, real and complex of $z^3 = 216i$. Write your answers in polar form.

QUESTION NINE

- (a) Describe the locus given by $|z - 1 + i| = 3$.

- (b) Find the equation of the locus described by $|z - 2i| = |z - 4|$.

Answers – Readiness Check

- Question One:**
- (a) $2.24 \operatorname{cis} 2.68$
 - (b) $3.61 \operatorname{cis} 0.588$
 - (c) $-5 + 3i$
 - (d) $-3 - 5i$
 - (e) $(-2 + i)(3 - 2i) = -4 + 7i$
 - (f) $\frac{3-2i}{-2+i} \times \frac{-2-i}{-2-i} = \frac{-8+i}{5}$

- Question Two:**
- (a) $4.33 - 2.5i$
 - (b) $10 \operatorname{cis} \frac{5\pi}{24}$
 - (c) $\frac{2}{5} \operatorname{cis} \frac{13\pi}{24}$
 - (d) $4096 \operatorname{cis} \frac{9\pi}{2} = 4096i$

- Question Three:**
- (a) $|z| = 2\sqrt{2} \quad \arg z = \frac{\pi}{4}$
 - (b) $|z| = 2 \quad \arg z = \frac{-\pi}{6}$

- Question Four:**
- (a) $8 + 4\sqrt{2} - 6\sqrt{2} - 6 = 2 - 2\sqrt{2}$
 - (b) $\frac{4-3\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{8+4\sqrt{2}-6\sqrt{2}+6}{2} = 7 - 5\sqrt{2}$

- Question Five:**
- (a) $(3\sqrt{2} - 2\sqrt{2})^2 = 2$
 - (b) $\frac{\sqrt{27k^2}}{\sqrt{12k^2}} = \frac{3\sqrt{3}k}{2\sqrt{3}k} = \frac{3}{2}$

- Question Six:**
- (a) $4(x+2) = (2x)^2$
 $4x^2 - 4x - 8 = 0$
 $4(x^2 - x - 2) = 0$
 $x = 2, -1$
 Only solution is $x = 2$

- (b) $x + 4 = x - 2k\sqrt{x} + k^2$
 $2k\sqrt{x} = k^2 - 4$
 $\sqrt{x} = \frac{k^2 - 4}{2k}$
 $x = \frac{(k^2 - 4)^2}{4k^2}$

- Question Seven:**
- (a) $z = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(22)}}{2}$
 $z = 4 \pm \sqrt{6}i$
 - (b) At $x = -4$, remainder = 6
 - (c) At $x = 1$, $2 - 1 + 3 + k = 0$
 and $k = -4$
 - (d) $(1+i)^3 - 6(1+i)^2 + 10(1+i) + k = 0$
 $-2 + 2i - 12i + 10 + 10i + k = 0$
 $k = -8$
 Solving $z^3 - 6z^2 + 10z - 8 = 0$
 gives $z = 1 + i$, $1 - i$ and 4

- Question Eight:**
- (a) $z^4 = -36$
 $r^4 \operatorname{cis} 4\theta = 36 \operatorname{cis} \pi$
 $r^4 = 36$ so $r = \sqrt{6}$
 $4\theta = \pi, 3\pi, 5\pi, 7\pi$
 $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
 $z_1 = \sqrt{6} \operatorname{cis} \frac{\pi}{4}$
 $z_2 = \sqrt{6} \operatorname{cis} \frac{3\pi}{4}$
 $z_3 = \sqrt{6} \operatorname{cis} \frac{5\pi}{4}$
 $z_4 = \sqrt{6} \operatorname{cis} \frac{7\pi}{4}$

Answers – Readiness Check

Question Eight: (b) $r^3 \operatorname{cis} 3\theta = 216 \operatorname{cis} \frac{\pi}{2}$

$$r^3 = 216 \text{ so } r = 6$$

$$3\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$z_1 = 6 \operatorname{cis} \frac{\pi}{6}$$

$$z_2 = 6 \operatorname{cis} \frac{5\pi}{6}$$

$$z_3 = 6 \operatorname{cis} \frac{3\pi}{2}$$

Question Nine: (a) Locus is a circle with centre (1, -1) and radius 3.

(b) $|x + iy - 2i| = |x + iy - 4|$
 $\sqrt{x^2 + (y - 2)^2} = \sqrt{(x - 4)^2 + y^2}$

$$x^2 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2$$

$$4y = 8x - 12$$

$$y = 2x - 3$$